MULTIMEDIA UNIVERSITY

**STUDENT ID NO**

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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

**TRIMESTER 1, 2016/2017**

## ETM7166 – DIGITAL SIGNAL PROCESSING SYSTEMS AND DESIGN IN TELECOMMUNICATIONS

(All sections / Groups)

22 OCTOBER 2016
2.30 p.m. – 5.30 p.m.
(3 Hours)

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**INSTRUCTION TO STUDENT**

1. This question paper consists of 12 pages including the cover page and Appendix.
2. There are **FOUR** questions in this paper. Attempt **ALL** questions.
3. All questions carry equal marks and the distribution of the marks for each question is given in square brackets.
4. The Appendix section contains useful tables and formulae.
5. Please write all your answers in the answer booklet provided. Number your questions clearly.

## Question 1

(a) Describe the phenomenon of aliasing in the time and frequency domains, and suggest how aliasing can be avoided.

[4 marks]

(b) For the signals $x[n]=\{\underline{4}, \quad 2, \quad 1, \quad 2, \quad 4\}$ and $h[n]=\{\underline{1}, \quad 3, \quad 3, \quad 1\}$, determine the following:

i. $3x[n]-2h[-n+3]$

[2 marks]

ii. Linear convolution of $x[n]$ and $h[n]$

[2 marks]

iii. 5-point circular convolution of $x[n]$ and $h[n]$

[2 marks]

iv. Auto-correlation of $h[n]$

[2 marks]

(c) A linear time-invariant (LTI) system is characterized by the system function:

$$H(z)=\frac{3z^{-1}}{1+0.3z^{-1}-0.18z^{-2}}$$

i. Determine the different possible regions of convergence (ROC) for the system.

[2 marks]

ii. If the system is realizable, determine the impulse response and the ROC of the system.

[4 marks]

(d) In signal processing, several techniques can be used to transform a signal from the time domain into the frequency domain.

i. Describe the relation between the Fourier Series, Fourier Transform, Discrete-Time Fourier Transform and Discrete Fourier Transform.

[2 marks]

ii. How do the above 4 techniques relate to the z-Transform and Laplace Transform?

[2 marks]

iii. Discuss the Short-Term Fourier Transform and explain its advantage and disadvantage. Suggest a technique that can be used to overcome the disadvantage suffered by Short-Term Fourier Transform.

[3 marks]

**Continues...**

## Question 2

(a) A finite impulse response (FIR) filter has the following coefficients:

$$h[n]=\left\{\underset{\uparrow}{1}, \quad -2, \quad 4, \quad -8, \quad 16, \quad -16, \quad 8, \quad -4, \quad 2, \quad 1\right\}$$

i. Suggest whether the FIR filter belong to Type I, II, III or IV.

[2 marks]

ii. Determine its frequency response $H\left(e^{j\omega}\right)$.

[3 marks]

iii. Derive the magnitude and phase equations of the filter.

[2 marks]

(b) Consider the following specifications for an FIR low-pass filter:

$$0.985 \leq |H(e^{j\omega})| \leq 1.015 \qquad 0 \leq |\omega| \leq 0.3\pi$$
$$|H(e^{j\omega})| \leq 0.015 \qquad 0.35\pi \leq |\omega| \leq \pi$$

i. List the different types of windows that can be used to meet the above specifications.

[2 marks]

ii. Design a linear phase FIR filter using one of the window types you mentioned in (b)(i). Justify your window selection.

[5 marks]

iii. Compute the filter order required to meet the specifications if you use other types of window you mentioned in (b)(i). Comment on the result.

[3 marks]

(c) Using bilinear transformation, design a second-order low-pass Butterworth filter that has a 3-dB cutoff frequency $\omega_c = \pi/5$.

[4 marks]

(d) For the two filters designed in (b) and (c), draw the block diagram or the signal flow graph of each filter in canonic form.

[4 marks]

**Continues...**

## Question 3

(a) An interpolator structure is shown in Fig. Q3 with $H(z)$ having a transfer function of $H(z) = z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 5z^{-5} + 6z^{-6} + 7z^{-7} + 8z^{-8}$.


Fig. Q3

i. Derive the 3-branch polyphase decomposition of $H(z)$ for the above system. Show explicitly the transfer function of each subfilter $E_m(z)$

[3 marks]

ii. Realize the system in Fig. Q3 using the 3-branch polyphase decomposition derived in Q3(a)(i).

[2 marks]

(b) The sampling rate of a signal is to be reduced from 18kHz to 300Hz by using a decimator. The specifications for the decimation filter $H(z)$ are as follows:

- Passband edge frequency, $F_p = 200Hz$
- Stopband edge frequency, $F_s = 300Hz$
- Passband ripple, $\delta_p = 0.02$
- Stopband ripple, $\delta_s = 0.01$

i. Determine the computational complexity, in terms of the number of multiplications per second, of the single-stage decimator.

[3 marks]

ii. Using $H(z) = F(z)G(z^{20})$, compute the computational complexity of the system using a two-stage decimator.

[6 marks]

(c) The Wiener filter is an optimal filter design based on a-priori statistical information.

i. Explain the underlying concept of Wiener filter.

[3 marks]

ii. An alternative to Wiener filter in solving the filter coefficients is the method of steepest descend. Explain how this method differs from Wiener filter.

[3 marks]

**Continues...**

(d) The transmitted data in communication systems are distorted most seriously by inter-symbol interference. Adaptive equalization method is often being employed to combat this form of interference. Explain how an adaptive equalization filter works.

[5 marks]

**Question 4**

(a) One main problem for telephone communications is the presence of echo in the network.

i. Explain the causes of echo in the telephone network.

[5 marks]

ii. Echo suppressor and echo canceller are the two techniques employed to overcome the detrimental effects of echo. With suitable diagrams, briefly describe the working principles behind both of these techniques

[10 marks]

(b) Digital speech signal is used to represent human conversation over the telecommunications network.

i. State four advantages of digital speech.

[4 marks]

ii. Speech coding is a technique used to encode digitized speech signal. State and explain the three gains that can be realized by using speech coding on digitized speech.

[6 marks]

**End of Questions**

# Appendix: Formula Sheet

## The $z$-transform

### Properties of the $z$-transform

| Property | $x[n]$ | $X(z)$ | $\mathcal{R}_x$ |
|---|---|---|---|
| Linearity | $ax[n]+by[n]$ | $aX(z)+bY(z)$ | $\mathcal{R}_x \cap \mathcal{R}_y$ |
| Time shifting | $x[n-m]$ | $z^{-m}X(z)$ | $\mathcal{R}_x$ |
| Time reversal | $x[-n]$ | $X(z^{-1})$ | $1/\mathcal{R}_x$ |
| Convolution | $x[n]*y[n]$ | $X(z)Y(z)$ | $\mathcal{R}_x \cap \mathcal{R}_y$ |

### Common $z$-transform pairs

| $x[n]$ | $X(z)$ | $\mathcal{R}_x$ |
|---|---|---|
| $\delta[n]$ | $1$ | $\forall z$ |
| $\delta[n-n_0]$ | $z^{-n_0}$ | Possibly $\forall z$ |
| $u[n]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert > 1$ |
| $-u[-n-1]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert < 1$ |
| $a^n u[n]$ | $\frac{1}{1-az^{-1}}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $-a^n u[-n-1]$ | $\frac{1}{1-az^{-1}}$ | $\lvert z\rvert < \lvert a\rvert$ |
| $na^n u[n]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert > \lvert a\rvert$ |
| $-na^n u[-n-1]$ | $\frac{az^{-1}}{(1-az^{-1})^2}$ | $\lvert z\rvert < \lvert a\rvert$ |

### Closed-form Expression for Some useful Series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a|<1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a|<1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

# FIR Filter Design

Ideal bandpass

$$h[n] = \frac{w_2}{\pi} \operatorname{sinc}\left(\frac{w_2(n-M/2)}{\pi}\right) - \frac{w_1}{\pi} \operatorname{sinc}\left(\frac{w_1(n-M/2)}{\pi}\right), \quad n = 0, 1, \ldots, M$$

Fixed windows

| Window | Window function |
|---|---|
| Rectangular | $w[n] = 1$ |
| Hann | $w[n] = 0.5 - 0.5\cos(2\pi n/M)$ |
| Hamming | $w[n] = 0.54 - 0.46\cos(2\pi n/M)$ |
| Blackman | $w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$ |

| Window | Passband ripple $20\log_{10}\delta_p$ | Stopband attenuation $20\log_{10}\delta_s$ | Transition width $\lvert w_p - w_s\rvert$ |
|---|---|---|---|
| Rectangular | $-13$ | $-21$ | $1.8\pi/M$ |
| Hann | $-31$ | $-44$ | $6.2\pi/M$ |
| Hamming | $-41$ | $-53$ | $6.6\pi/M$ |
| Blackman | $-57$ | $-74$ | $11\pi/M$ |

Kaiser window

$$w[n] = \frac{I_0\left(\beta(1-(n/\alpha-1)^2)^{0.5}\right)}{I_0(\beta)}$$

$$\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$$

$$M = \begin{cases} (A-7.95)/(2.285\Delta w), & A \ge 21 \\ 5.655/\Delta w, & A < 21 \end{cases}$$

Optimal Filter Design (Parks-McClellan Algorithm) Estimated Filter Order

$$N = \frac{-20\log\sqrt{\delta_p\delta_s} - 13}{14.6\Delta f}$$

**CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS**

| | $h[n]$ symmetric: $h[n] = h[N\text{-}n]$ | $h[n]$ antisymmetric: $h[n] = -h[N\text{-}n]$ |
|---|---|---|
| $N$ even | **Type I Linear Phase Filter**<br>$H(e^{j\omega}) = e^{-jN\omega/2}\sum_{k=0}^{N/2} a[k]\cos(k\omega)$<br>$a[0] = h[N/2]$<br>$a[k] = 2h[(N/2)-k]$ | **Type III Linear Phase Filter**<br>$H(e^{j\omega}) = je^{-jN\omega/2}\sum_{k=1}^{N/2} c[k]\sin(k\omega)$<br>$c[k] = 2h[(N/2)-k]$ |
| $N$ odd | **Type II Linear Phase Filter**<br>$H(e^{j\omega}) = e^{-jN\omega/2}\sum_{k=1}^{(N+1)/2} b[k]\cos((k-1/2)\omega)$<br>$b[k] = 2h[\frac{(N+1)}{2} - k]$ | **Type IV Linear Phase Filter**<br>$H(e^{j\omega}) = je^{-jN\omega/2}\sum_{k=1}^{(N+1)/2} d[k]\sin((k-1/2)\omega)$<br>$d[k] = 2h[\frac{(N+1)}{2} - k]$ |

## IIR Filter Design

Normalized Butterworth lowpass

| $N$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.000 | | | | | | | |
| 2 | 1.414 | 1.000 | | | | | | |
| 3 | 2.000 | 2.000 | 1.000 | | | | | |
| 4 | 2.613 | 3.414 | 2.613 | 1.000 | | | | |
| 5 | 3.236 | 5.236 | 5.236 | 3.236 | 1.000 | | | |
| 6 | 3.864 | 7.464 | 9.142 | 7.464 | 3.864 | 1.000 | | |
| 7 | 4.494 | 10.10 | 14.59 | 14.59 | 10.10 | 4.494 | 1.000 | |
| 8 | 5.126 | 13.14 | 21.85 | 25.69 | 21.85 | 13.14 | 5.126 | 1.000 |

Filter order

$$d = \left( \frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right)^{0.5}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

| Design | Filter order |
|---|---|
| Butterworth | $N \geq \frac{\log d}{\log k}$ |
| Chebyshev I, II | $N \geq \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$ |
| Elliptic | $N \geq \frac{\log(16/d^2)}{\log(2/u)}$ |
| | $u = \frac{1-(1-k^2)^{0.25}}{1+(1-k^2)^{0.25}}$ |

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

Frequency transformations

| Target class | Transformation | Edge frequencies of target class |
|---|---|---|
| Highpass | $s \rightarrow \dfrac{\Omega_p \Omega'_p}{s}$ | $\Omega'_p$ |
| Bandpass | $s \rightarrow \Omega_p \dfrac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$ | $\Omega_l, \Omega_u$ |
| Bandstop | $s \rightarrow \Omega_p \dfrac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$ | $\Omega_l, \Omega_u$ |

Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

Bilinear transformation

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T_s}\frac{1-z^{-1}}{1+z^{-1}}}$$

$$\Omega = 2\tan(w/2)/T_s$$

# Discrete-time Fourier Analysis

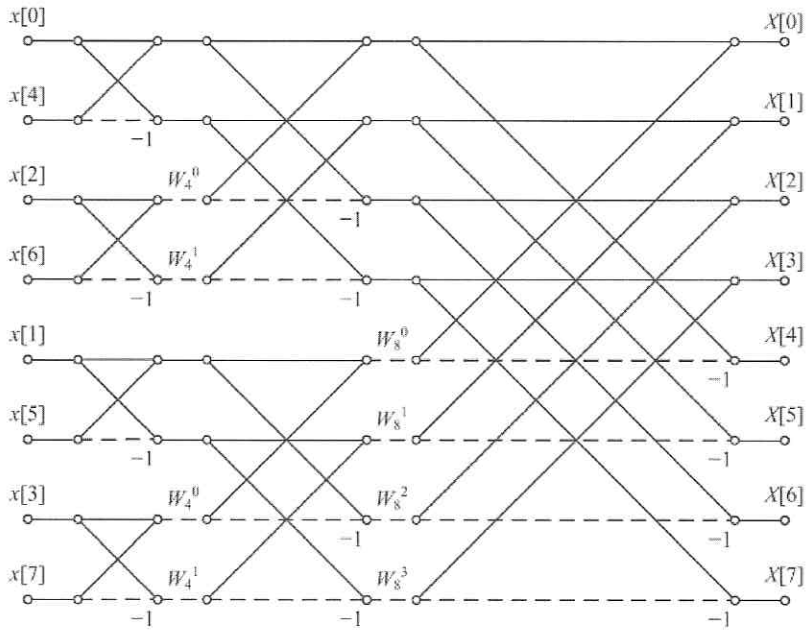
The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

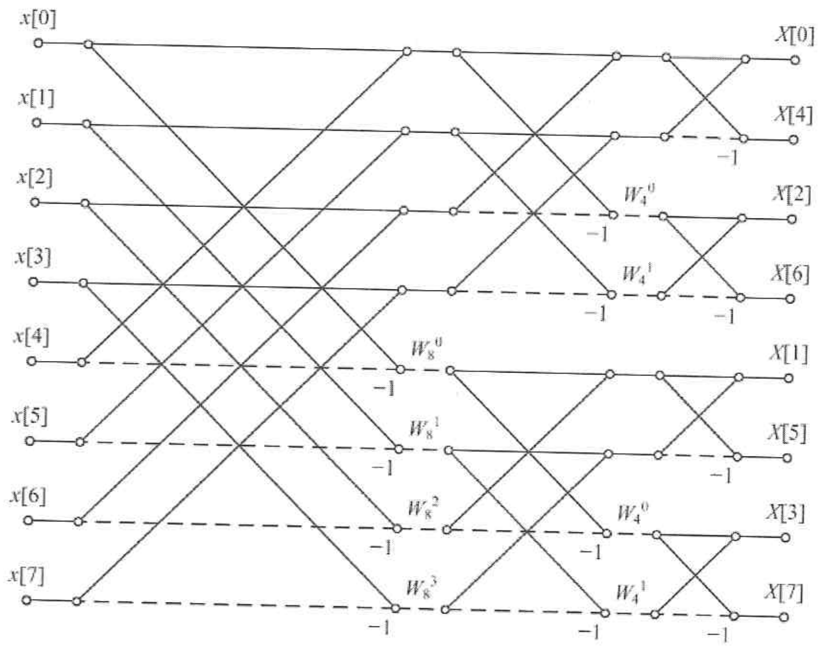
Properties of the DFT

| Property | $x[n]$ | $X[k]$ |
|---|---|---|
| Linearity | $A_1x_1[n] + A_2x_2[n]$ | $A_1X_1[k] + A_2X_2[k]$ |
| Time shifting | $x[\langle n - n_0\rangle_N]$ | $X[k]W_N^{kn_0}$ |
| Frequency shifting | $x[n]W_N^{-k_0n}$ | $X[\langle k - k_0\rangle_N]$ |
| Time reversal | $x[\langle -n\rangle_N]$ | $X[\langle -k\rangle_N]$ |
| Conjugation | $x^*[n]$ | $X^*[\langle -k\rangle_N]$ |
| Convolution | $x[n] \circledast y[n]$ | $X[k]Y[k]$ |
| Modulation | $Nx[n]y[n]$ | $X[k] \circledast Y[k]$ |

The decimation-in-time FFT



The decimation-in-frequency FFT




**End of Paper**